Numerical study of acoustic waves around and inside an underground cavity

CTBTO Young Scientist Research Award 2014
WWTF project 2015

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Wave animation

Scattered wave field

Zoom
Objectives

- Contribution to the **scientific groundwork** for OSI

- Numerical investigation
  - Development of new method for elastic-acoustic problems
  - Improvement of computational algorithms

- Applied investigation
  - Define characteristics of wave field interactions
  - Improve and design methods and experiments
# On-Site Inspection

## Request of OSI

### Launch

#### Equipment - Training/Field tests - OSI Action Plan

<table>
<thead>
<tr>
<th>Techniques</th>
<th>IFE08</th>
<th>IFE14</th>
<th>Techniques</th>
<th>IFE08</th>
<th>IFE14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual Observations</td>
<td>●</td>
<td>●</td>
<td>Environmental measurements</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Seismological aftershock monitoring</td>
<td>●</td>
<td>●</td>
<td>Electrical conductivity measurements</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Gamma radiation monitoring</td>
<td>●</td>
<td>●</td>
<td>Air-borne gamma spectroscopy</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Magnetic field mapping</td>
<td>●</td>
<td>●</td>
<td>Active seismic survey</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Gravitational field mapping</td>
<td>●</td>
<td>●</td>
<td>Resonance seismometry</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Multi-spectral imaging</td>
<td>●</td>
<td>●</td>
<td>Drilling</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Argon-37 and radioxenon measurement</td>
<td>●</td>
<td>●</td>
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</tbody>
</table>

Key: ● Tested, ○ Partly tested, ● Not tested
Seismic Methods

Subsurface structure after an UNE

(Adushkin & Spivak, 2004)

Simplified model

Sources:
- Passive seismic sources: Body waves, Surface waves, Noise
- Active seismic sources: Vibroseis, Explosions
Underling physics

- **Acoustic wave equation** in medium $\Omega_1$

- **Elastic wave equation** in medium $\Omega_2$

- Time harmonic cases:
  - Spherical point source
  - Plane wave sources
Underling physics

- **Acoustic wave equation** in medium $\Omega_1$

- **Elastic wave equation** in medium $\Omega_2$
  $\Rightarrow$ Focus on compressional waves

- Time harmonic cases:

  Spherical point source

  Plane wave sources
In/Out Scattering

Separation into the incident and scattered wave field:

\[ p = p_{inc} + p_{scat} \]

⇒ Better inside to the wave field interaction
Numerical results (1)

- **Scattered wave field** from a plane wave from the bottom left with \( f = 16\text{Hz} \)
- Cavity at depth \( D = 600\text{m} \) and diameter \( d = 60\text{m} \)
- Material parameters:
  \[ v_1 = 300\text{m/s}, \rho_1 = 1\text{kg/m}^{-3} \]
  \[ v_2 = 3000\text{m/s}, \rho_2 = 1000\text{kg/m}^{-3} \]
Numerical results (1)

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### Netgen/NGsolve
- Automatic 2D/3D adaptive mesh generator
- High-order Finite Element Method
- Applicable for Heat-flow, Acoustics, Elasticity, Navier-Stokes, Maxwell, etc.
- Open-source, C++, MPI – parallel, Python Plugin
Numerical Results (2)

- Scattering profile
- No surface

- Plane wave from the bottom with $f = 16\text{Hz}$
- Cavity at depth $D = 600\text{m}$ and diameter $d = 60\text{m}$
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  \[ v_1 = 3000\text{m/s}, \rho_1 = 1000\text{kg/m}^3, \quad v_2 = 300\text{m/s}, \rho_2 = 1\text{kg/m}^3 \]
Numerical Results (2)

- Scattering profile
- No surface

![Total field and Scattered field images]

- Plane wave from the bottom with $f = 16\text{Hz}$
- Cavity at depth $D = 600\text{m}$ and diameter $d = 60\text{m}$
- Material parameters:
  \[ \nu_1 = 3000\text{m/s}, \rho_1 = 1000\text{kg/m}^3, \nu_2 = 300\text{m/s}, \rho_2 = 1\text{kg/m}^3 \]
Express the incident and the scattered wave field by linear combinations of **spherical vectors** and solve a system of linear equations

\[
U_0 = \sum_{l \geq 0} \{ j_{l+1}(\omega \alpha_2 r) Y_{l0}^+ - j_{l-1}(\omega \alpha_2 r) Y_{l0}^- \} \exp \{-i[\pi/2(l + 1)]\}
\]

\[
U_1 = \sum_{l \geq 0} \left\{ a_l^{(1)} j_{l+1}(\omega \alpha_1 r) + [b_l^{(1)}]_{l+1}(\omega \beta_1 r) \right\} Y_{l0}^+ \\
+ \left[ -a_l^{(1)} j_{l-1}(\omega \alpha_1 r) + (l + 1) b_l^{(1)} j_{l-1}(\omega \beta_1 r) \right] Y_{l0}^- \exp \{-i[\pi/2(l + 1)]\}
\]

\[
U_2 = \sum_{l \geq 0} \left\{ a_l^{(2)} h_{l+1}(\omega \alpha_2 r) + [b_l^{(2)}]_{l+1}(\omega \beta_2 r) \right\} Y_{l0}^+ \\
+ \left[ -a_l^{(2)} h_{l-1}(\omega \alpha_2 r) + (l + 1) b_l^{(2)} h_{l-1}(\omega \beta_2 r) \right] Y_{l0}^- \exp \{-i[\pi/2(l + 1)]\}
\]

For each index \( l \) find the coefficients to match at the boundary \( r = R \)
Scattering cross sections
(Valeri Korneev, Geophys. J. Int. (1993))

- $S = E_{in}/E_{scat}$ . . . Ratio between scattered and incoming energy flow
- $kR = U/\lambda$ . . . Ratio between cavity circumference and incident wavelength

⇒ Modeling the cavity as an **acoustic inclusion** is crucial to resonance seismometry
Summary & Outlook

- Project distinguished by the Young Scientists Research Award
- Wave propagation with focus on acoustic case
- Identification of underlying physics
- Investigation of characteristics (analytically and numerically)
- Set up of an efficient numerical code
- Extension to full elastic wave field
- Extension to 3D
- Numerical improvement and Post-processing
- Incorporation of our findings into the practices of an OSI
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Thank you!
In/Out Separation

- Ansatz \( p = p_{inc} + p_{scat} \)
- \( p_{inc} \) solves
  \[
  \nabla^2 p_{inc} + k_1 p_{inc} = \delta(x - x_0) + \text{rad. cond. for } \|x - x_0\| \to \infty
  \]
- The remaining unknown \( p_{scat} \) is then the solution of
  \[
  \nabla \cdot \left( \frac{1}{\rho_1} \nabla p_{scat} \right) + \frac{k_1}{\rho_1} p_{scat} = 0 \quad \text{in } \Omega_2
  \]
  \[
  \nabla \cdot \left( \frac{1}{\rho_2} \nabla p_{scat} \right) + \frac{k_2}{\rho_2} p_{scat} = \frac{1}{\rho_2} (k_1 - k_2) p_{inc} \quad \text{in } \Omega_1
  \]
  \[
  p_{scat} = -p_{inc} \quad \text{on } \Gamma
  \]

  + radiation condition for \( r \to \infty \)
Numerical method

- High-Order Finite Element Method
  - Can handle highly complex, irregularly shaped geometries
  - Tends to exponentially increase the accuracy of the computations

- Software
  - **Netgen**: Automatic 2D/3D tetrahedral adaptive mesh generator
  - **Ngsolve**: Applicable for Heat-flow, Acoustics, Elasticity, Navier-Stokes, Maxwell, etc.
    - Open-source, C++, MPI – parallel, Python Plugin

- Hardware
  - Internal server with 48 cores
  - Vienna Scientific Cluster 3 - 140/2020 dual nodes à 8 cores