1. Introduction

Inverse modeling is a formal approach for estimation of uncertain parameters of a system using inversion of relevant observed data. In atmospheric modeling it is successfully applied to estimation of a source term given measurements. It is based on systematic comparison of measured data with simulation results based on different source hypotheses. Let \( x \) be a vector of source term elements denoted by their spatial-temporal characteristic and \( y \) a vector of observations. They can be related via an atmospheric transport model (ATM), \( y(x) \) as \( y = f(x) \). The goal is to find such a \( x \) that the deviation \( \Delta = f(x) - y \) is minimal. Lin-ear model can be represented by a source-receptor-sensitivity (SRS) matrix \( M \). Because of errors in both \( x \) and \( y \) the exact match is never reached. However, we can quantify these errors (assign distributions to them) and treat them in a statistically consistent manner. This must be given a priori.

We have already applied a formal inverse modeling method to the Xe-133 observations in April 2013. We used a broader set of samples including also non-detections from 6 sampling stations in total (JPX38, RJX58, CNX22, CNX20, MNX45 and USX77). Now, the method will be applied to Xe-131m measurements. In addition to using the detections and non-detection at JPX38, we will also try to include non-detections from 6 sampling stations in total (JPX38, RJX58, CNX22, CNX20, MNX45 and USX77).

A source estimate is obtained by optimizing an objective function (cost function) in a way that the discrepancy between model simulations and samples is minimized on one hand while keeping the estimate compatible with its a priori value on the other hand. Since the cost function expresses agreement of samples with a release originated from a given source location, it can be used for identification of the most plausible source regions in cases where this is not known or uncertain.

2. Inversion methodology

Foundation for the inverse modeling stems from the Bayes’ theorem. Let \( p(x) \) and \( p(y) \) be probability density functions (pdfs) of \( x \) and \( y \) respectively. The goal is to get posterior pdf \( p(y|x) \). Optimum \( x \) is then maximum of this posterior (maximum a posteriori estimate)

\[
\hat{x} = \text{arg max} p(y|x(x))
\]

If both likelihood function \( p(y|x) \) and prior \( p(x) \) are Gaussian pdfs,

\[
p(y|x) = N(Mx + R, \Sigma) \quad p(x) = N(0, \Sigma_0)
\]

then maximizing \( p(y|x) \) is equivalent to

\[
x = \Sigma_0^{-1} (y - Mx)
\]

Here, \( R \) and \( B \) represent error covariances of observations and source term prior (first guess). Both these matrices are usually assumed to be diagonal. \( R \) contains not only measurement error but it should con-tain also a model error. Optimal source \( x \) is found via minimizing cost function

\[
f(x) = (x - \hat{x})^T \Sigma_0^{-1}(x - \hat{x})
\]

The first term on r.h.s. measures deviations of model predictions from observations and the second term acts as a regularization and measures deviation from source hypotheses prior \( x_0 \). Minimization can be done analytically by and leads to a system of linear equations for \( x \) (after subtracting \( x_0 \)).

\[
(M^T \Sigma_0^{-1} M + R) \hat{x} = M^T \Sigma_0^{-1} y
\]

Posterior error covariance matrix \( \Sigma_0 \)

\[
\Sigma = (M^T \Sigma_0^{-1} M + R)^{-1}
\]

Suppression of possible negative emissions is done by repetitive solution of LSE with increased regularization on prior which prevents probabilistic elements of the solution to diverge from (non-negative) \( x_0 \).

3. Data

- We aim to retrieve a source location/source term using Xe-131m/Xe-133 samples from stations JPX38 (Takasaki) and RJX58 (Ussuriysk).
- Previously we already did a similar study using 36 Xe-133 samples (including non-detection) from 6 stations.
- In this study we use only 9 samples — 5 from JPX38 and 4 from RJX58, see Figure 1.
- Before inversion, background (6 months average without samples used) was subtracted from samples.

4. Experiment

- Atmospheric transport modeling:
  - We assume point releases only (from a single grid cell containing DPRK test site in the case of know source location — explicit a-priori knowledge)
  - SRS calculated using backward runs of FLEXPART 9.0
  - 10 samples — 10 runs back in time until Apr 06 2013, each with 2 million particles
  - SRS calculations performed with high accuracy
  - ECMWF input data 0.25° horizontal resolution, 91 vertical levels, 3 hour temporal resolution, convection enabled in FLEXPART
  - FLEXPART output on lon-lat grid with dx = 0.25° and dy = 0.2° every 3 hours
- We assume 3 vertical layers in order to account for complex terrain at the DPRK test site which varies between 500 and 2200 m asl 0–100, 100–500 and 500–1000 meters above model ground, which is 880 – 1500 m above sea level — we need to calculate SRS of all samples with all 3 time elements of each layer
- First guess: \( x_0 \) = 0 \( B_{rx} = 1 \times 10^{-1} \text{Bq per element of solution vector} \)
- Inversion setup:
  - Estimation of emission in time from all vertical layers simultaneously (dim x(a204))
  - Model error assumed to be proportional to measurements with an offset added to measurement variance:
    - \( R = \text{diag}(0.2 y + 1 \times 10^{-3} y^2 + 0.5 \text{I}) \text{Bq per vector of measurement errors obtained from VODEC} \)
  - Samples were decay-corrected to Apr 06 2013 00:00

5. Results: Known source location

- Inversion procedure was applied to all possible source grid cells in our domain
- Xe-131m samples were used (5 from JPX38 and 4 from RJX58)
- Source term profiles obtained using Xe-133 and Xe-131m samples. Timing corresponds to release at Apr 6 around 12:00
- Source strength of Xe-133 approx 10^4 higher than with broader set of samples including non-detection from more stations
- Source strength of Xe-133 approx 10^6 higher than with broader set of samples including non-detection from more stations
- Timing of retrieved source terms is similar, the main peak corresponds to release at April 6 around 12:00
- In the future, we can try inversion using both isotopes simultaneously assuming their expected ratios

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Conclusion

- Source strength of Xe-133 approx 10^6 higher than with broader set of samples including non-detection from more stations
- Timing of retrieved source terms is similar, the main peak corresponds to release at April 6 around 12:00
- In the future, we can try inversion using both isotopes simultaneously assuming their expected ratios